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A Further Study on the Ideal Ramp Filter of Analytical Image Reconstruction

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Abstract: The ideal ramp filter is a generalized function defined by the inverse Fourier transform. Designing practical digital filters of the ideal ramp filter is the most important task. The equation of the unit impulse response of the ideal ramp filter was deduced. Two design patterns of practical digital filters were proposed, which are continuous pattern and discrete pattern. Three existing practical filters were analyzed. Two new practical filters were designed. A comparison of the five filters was made through a simulation experiment. In general, the discrete methods coming from the discrete design pattern are better than the continuous methods coming from the continuous design pattern. Between the two discrete methods, the infinite length discrete method is better than the adaptable length discrete method. Using them, better reconstruction quality can be obtained. Among the three continuous methods, the linear method is the best, which is acceptable in engineering applications. The constant method and the parabola method cannot make good effect in image reconstruction.

Key words: CT; image reconstruction; ideal ramp filter; filter design; practical filter

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CT (Computed Tomography) technology is one of the best non-destructive test technologies. It has been applied in medical fields and industrial fields successfully. X-CT is the main form of CT, which obtain projection data by using detector to detect X-ray's attenuation. In industrial X-CT, the CT algorithms can be divided into two methods, analytical methods and algebraic methods[1,2]. The most popular method for analytical image reconstruction remains the filtered back projection (FBP) algorithm. For example, the FDK algorithm and the Katsevich algorithm are both belong to the FBP type algorithms[3,4]. So, if one wants to research some basic questions of a FBP type algorithm, one should research them by using the parallel beam FBP algorithm for it is a basic and classical algorithm.

FBP algorithm consists of a filtering step and a back projection step. There are many factors that can impact the reconstructed image's quality such as interpolation method, ramp filter, projection numbers, projection sampling interval, scattering of X-ray, beam hardening etc[5]. Among them, the design and implementation of the ramp filter is a very important factor which can impact the CT images' quality considerably. However, the filters' design details are considered as secrets, for the work is hard and important. Therefore, there are many questions about the ramp filter which are deserved to be researched.

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In 2005, Wei Yuchuan, Wang Ge, and Jiang Hsieh published their paper “An Intuitive Discussion on the Ideal Ramp Filter in Computed Tomography (I)”[6]. It is the first paper that proposed the concept of ideal ramp filter and deduced the formula of the unit impulse response $h(t)$ of the ideal ramp filter. In the paper, the authors also gave three practical filters as approximate realizations of the ideal ramp filter. The paper implied that the CT image reconstructed with the third practical filter is better than the image with the first or the second practical filter, and suggested that there exist other similar filters. Thus it is reasonable and necessary for us to design more filters, compare their performance carefully, find out the better ones and explain the reasons.

Based on their paper’s research, the paper will deduce the formula of $h(t)$ of the ideal ramp filter more detailed and more clearly and propose design patterns of the practical filters and design two new practical filters. Then the five practical filters (two new filters and three filters designed by the paper[6]) will be applied in a simulation experiment to compare their properties.

1 Deduction of the ideal ramp filter and design patterns of practical filters

In FBP algorithm, the ramp filter’s frequency characteristic is

$$H(\omega)=|\omega| \tag{1}$$

Thus, the unit impulse response is

$$h(t)=\int_{-\infty}^{\infty}|\omega|e^{j2\pi\omega t}\mathrm{d}\omega \tag{2}$$

For

$$\int_{-\infty}^{\infty}|\omega|\,\mathrm{d}\omega\to\infty \tag{3}$$

$h(t)$ does not exist in the traditional sense[7]. A usual method is to give $H(\omega)$ a window to limit its bandwidth. Given a rectangle window, it becomes R-L filter. Given a “sinc” window, it becomes S-L filter. They are the most popular two filters which have been applied into mature CT systems.

Different from the “window adding” method in Fourier domain, Wei’s paper considered how to obtain the ideal ramp filter $h(t)$ in the real space.

$$\begin{aligned}h(t)&=\int_{-\infty}^{\infty}|\omega|e^{j2\pi\omega t}\mathrm{d}\omega\\&=\int_{-\infty}^{\infty}\omega\,\mathrm{sgn}(\omega)e^{j2\pi\omega t}\mathrm{d}\omega\end{aligned} \tag{4}$$

In (4),

$$\mathrm{sgn}(\omega)=\begin{cases}1 & \omega>0\\ 0 & \omega=0\\ -1 & \omega<0\end{cases} \tag{5}$$

Let ω is a constant and t is a variable, then

$$\frac{d}{\mathrm{d}t}\left(\frac{1}{j2\pi}e^{j2\pi\omega t}\right)=\omega e^{j2\pi\omega t} \tag{6}$$

Then (4) can be

$$h(t)=\frac{1}{j2\pi}\frac{d}{\mathrm{d}t}\left(\int_{-\infty}^{\infty}\mathrm{sgn}(\omega)\, e^{j2\pi\omega t}\mathrm{d}\omega\right) \tag{7}$$

For

$$F\left(\frac{j}{\pi t}\right)=\mathrm{sgn}(\omega) \tag{8}$$

Here $F(\bullet)$ is a Fourier transform of a signal.
So (7) can be

$$h(t)=\frac{1}{j2\pi}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{j}{\pi t}\right)=\frac{1}{2\pi^2}\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{1}{t}\right)=\begin{cases}-\dfrac{1}{2\pi^2 t^2} & t\neq 0\\ \infty & t=0\end{cases} \tag{9}$$

Obviously, $h(t)$ is an ideal but not practical filter. For design some practical filters, we must find some method to evaluate $h(0)$ which will be named central value.

From the Fourier transform, we know

$$\int_{-\infty}^{\infty}h(t)\,\mathrm{d}t=H(0)=0 \tag{10}$$

This is a very important control condition for (9). Using (10) and some limit conditions, we can figure out some central value $h(0)$, thus some practical filters of the ideal ramp filter can be designed.

If the signal $h(t)$ is sampled using the sampling interval "d" which is also the projection's sampling interval, (9) can be

$$h(n)=\begin{cases}-\dfrac{1}{2\pi^2 n^2 d^2} & n\neq 0\\ \infty & n=0\end{cases} \tag{11}$$

(11) is also not a practical filter for $h(0)=\infty$, For design some practical filters, we must find

some method to evaluate $h(0)$ which will be named central value.

Similar to (10), (12) can be regarded as a discrete version of (10).

$$\sum_{n=-\infty}^{\infty} h(n) = 0 \tag{12}$$

This is a very important control condition for (11). Using (12) and some limit conditions, we can figure out some central value $h(0)$, thus some practical filters of the ideal ramp filter can be designed.

In sum, we can summarize two design patterns of practical filters of the ideal ramp filter.

(1) Continuous pattern

Just using (9) and (10), $h(0)$ cannot been calculated. We should consider there is a symmetrical curve whose definition domain is $[-\varepsilon, \varepsilon]$ and whose maximal value is $h(0)$. Thus, according to the control condition (10), a finite $h(0)$ can be calculated and a practical continuous filter can be designed. A practical digital filter $h(n)$ can be obtained through sampling the practical continuous filter $h(t)$.

(2) Discrete pattern

Different from the continuous pattern, just using (11) and (12), a finite central value $h(0)$ can be calculated. There are two techniques to calculate it. The first one is regarding $h(n)$ as an infinite length sequence, so the key question become if the infinite series $\sum_{n=1}^{\infty} h(n)$ and $\sum_{n=-\infty}^{-1} h(n)$ are convergent. The second one is regarding $h(n)$ as a finite length sequence, so according to (12), there is always a finite $h(0)$ and a practical discrete filter can be obtained.

2 Three practical filters proposed by WEI Yuchuan

In 2005, WEI Yuchuan proposed three practical filters and made a simulation experiment to validate their effectiveness. In the section, a concise introduction will be given and an analysis for the three practical filters will be given.

2.1 Constant filter (Filter 1)

This is the most straightforward approximation of $h(t)$. Its expression is

$$h_1(t) = \begin{cases} -\dfrac{1}{2\pi^2 t^2} & |t| \geq \varepsilon \\ \dfrac{1}{2\pi^2 \varepsilon^2} & |t| < \varepsilon \end{cases} \tag{13}$$

Similar to (10), exist

$$\int_{-\infty}^{\infty} h_1(t)\,\mathrm{d}t = 0 \tag{14}$$

(14) is a control condition of (13). In (13), ε is a sufficient small number. In the filter, a positive constant is used in the region $[-\varepsilon, \varepsilon]$ to approximate infinity in an infinitely small neighborhood. The constant $h_1(0)$ is calculated by (13) and (14).

The figure of (13) is shown as Figure 1(a). A remarkable characteristic is that it is discontinuous at $|t| = \varepsilon$.

For discretize $h_1(t)$ to construct a digital filter, let $\varepsilon = d$, and sample $h_1(t)$ with the sampling interval *d*, and we can get a digital filter. That is

$$h_1(n) = \begin{cases} -\dfrac{1}{2\pi^2 n^2 d^2} & n \neq 0 \\ \dfrac{1}{2\pi^2 d^2} & n = 0 \end{cases} \tag{15}$$

For $h_1(t)$ of $[-\varepsilon, \varepsilon]$ is a constant, we named the filter as constant filter.

2.2　Linear filter (Filter 2)

For improve the practical filter of (13) to remove the discontinuities at $|t| = \varepsilon$, a linear method was proposed.

The expression is

$$h_2(t) = \begin{cases} -\dfrac{1}{2\pi^2 t^2} & |t| \geq \varepsilon \\ \dfrac{1}{2\pi^2 \varepsilon^2}\left(3 - \dfrac{4|t|}{\varepsilon}\right) & |t| < \varepsilon \end{cases} \tag{16}$$

Its control condition is

$$\int_{-\infty}^{\infty} h_2(t)\,\mathrm{d}t = 0 \tag{17}$$

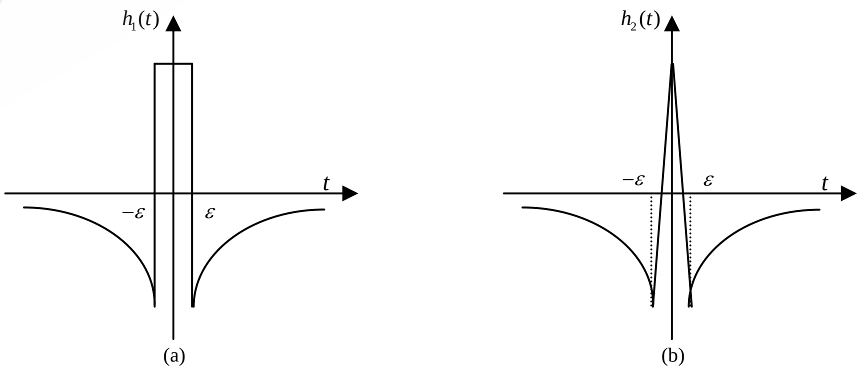

Fig.1　Two practical filters of the ideal ramp filter, (a) constant filter and (b) linear filter

The expression in the region $[-\varepsilon, \varepsilon]$ of $h_2(t)$ is calculated by the control of (17) to (16). The figure is shown as Figure 1(b). A remarkable characteristic is that it is continuous at $|t| = \varepsilon$.

For discretize $h_2(t)$ to construct a digital filter, let $\varepsilon = d$, and sample $h_2(t)$ with the sampling interval *d*, and we can get a digital filter. That is

$$h_2(n) = \begin{cases} -\dfrac{1}{2\pi^2 n^2 d^2} & n \neq 0 \\ \dfrac{3}{2\pi^2 d^2} & n = 0 \end{cases} \tag{18}$$

For $h_2(t)$ of $[-\varepsilon, \varepsilon]$ is a line, we named the filter as linear filter.

2.3 Infinite length discrete filter (Filter 3)

From 2.1 and 2.2, we know that the constant filter and the linear filter are all designed using the continuous pattern. If we consider from the discrete pattern angle, we can use (12) to control (11) and can obtaining a finite central value $h(0)$ and a practical digital filter is constructed.

WEI Yuchuan designed a practical digital filter according to the idea but did not give a strict calculation. Here, a detailed calculation is given.

The expression of the filter is

$$h_3(n) = \begin{cases} -\dfrac{1}{2\pi^2 n^2 d^2} & n \neq 0 \\ \dfrac{1}{6d^2} & n = 0 \end{cases} \tag{19}$$

The key question is that how to calculate the central value $h_3(0)$.

We know

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6} \tag{20}$$

So

$$\sum_{n=1}^{\infty}\left(-\frac{1}{2\pi^2 n^2 d^2}\right) = -\frac{\pi^2}{6} \times \frac{1}{2\pi^2 d^2} = -\frac{1}{12d^2}$$

And

$$\sum_{n=-\infty}^{-1}\left(-\frac{1}{2\pi^2 n^2 d^2}\right) = \sum_{n=1}^{\infty}\left(-\frac{1}{2\pi^2 n^2 d^2}\right) = -\frac{1}{12d^2}$$

So

$$\sum_{n\neq 0} h_3(n) = \sum_{n=-\infty}^{-1}\left(-\frac{1}{2\pi^2 n^2 d^2}\right) + \sum_{n=1}^{\infty}\left(-\frac{1}{2\pi^2 n^2 d^2}\right) = -\frac{1}{6d^2}$$

According to (12),

$$h_3(0) = -\sum_{n\neq 0} h_3(n) = \frac{1}{6d^2}$$

The calculation is completed.

For the filter is designed from the discrete angle and $h(n)$ is regarded as infinite, we named the filter as infinite length discrete filter.

2.4 WEI's three filters analyzing

As has been depicted in section 1, there are two design patterns of practical filters of the ideal ramp filter. One is from the continuous angle to calculate the central value. The other is from the discrete angle to calculate the central value.

The former two filters depicted in 2.1 and 2.2 respectively belong to the pattern 1. The latter one depicted in 2.3 belongs to the pattern 2.

The first is the constant filter, which is discontinuous at $|t| = \varepsilon$. The second is the linear filter, which is continuous at $|t| = \varepsilon$ and $t = 0$ but is not derivable at $t = 0$. A natural idea is that if we can obtaining higher performance filter through constructing a curve on $[-\varepsilon, \varepsilon]$ that is continuous at $|t| = \varepsilon$ and derivable at $t = 0$. According to the idea, the paper will design the type of practical digital filters.

The third is the infinite discrete filter. Seen from the discrete angle, it is an ideal method for it is related to all points of $h(n)$ that is an infinite length sequence. However, a computational filter's unit impulse response must be finite length. In a practical reconstruction calculation, we truncate the infinite discrete filter so that the ramp filter's length should be double of that of the projection signal.

So there is a consequent idea that we can calculate $h(0)$ by using (12) to control the specific length filter $h(n)$. Which one is better is an important problem that will be analyzed in section 4 and 5.

3 Two new practical digital filters

According to the two design patterns of practical filters, inspired by the three filters, two new practical digital filters will be designed. One is designed using the continuous pattern and the other is designed using the discrete pattern.

3.1 Parabola filter (Filter 4)

It is designed using the continuous pattern. The constant filter construct a constant on the region $[-\varepsilon, \varepsilon]$. The linear filter construct a linear symmetric curve on the region $[-\varepsilon, \varepsilon]$.

Obviously, we can construct a parabola on the region $[-\varepsilon, \varepsilon]$ (Figure 2).

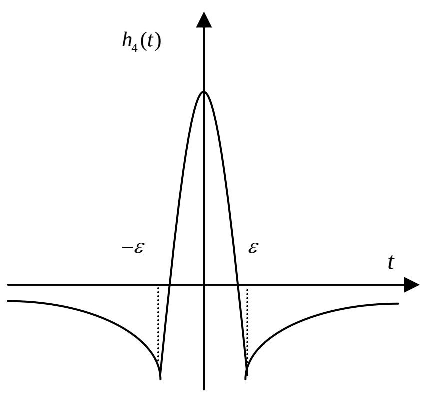

Fig.2 The parabola filter of the ideal ramp filter

Now the expression of $h_4(t)$ is unknown on the region $[-\varepsilon, \varepsilon]$, so we must using some condition to calculate the expression.

From Figure 2, we know the first condition is that the expression of the parabola can be supposed as

$$y = at^2 + c \tag{21}$$

The second condition is that point $\left(-\varepsilon, -\frac{1}{2\pi^2\varepsilon^2}\right)$ and $\left(\varepsilon, -\frac{1}{2\pi^2\varepsilon^2}\right)$ are on the parabola.

The third condition is

$$\int_{-\infty}^{\infty} h_4(t)\,\mathrm{d}t = 0 \tag{22}$$

Thus, according to condition 1 and condition 2, we can construct the first equation

$$-\frac{1}{2\pi^2\varepsilon^2} = a\varepsilon^2 + c \tag{23}$$

According to (22), we know

$$\int_0^{\varepsilon} \left(at^2 + c\right)\mathrm{d}t + \int_{\varepsilon}^{\infty} -\frac{1}{2\pi^2 t^2}\,\mathrm{d}t = 0 \tag{24}$$

Simplify (24), get

$$\frac{a}{3}\varepsilon^3 + c\varepsilon = \frac{1}{2\pi^2\varepsilon} \tag{25}$$

Combine (23) and (25), get an equation set.

$$\begin{cases} a\varepsilon^2 + c = -\dfrac{1}{2\pi^2\varepsilon^2} \\ \dfrac{a}{3}\varepsilon^3 + c\varepsilon = \dfrac{1}{2\pi^2\varepsilon} \end{cases} \tag{26}$$

Solve (26), get

$$\begin{cases} a = -\dfrac{3}{2\pi^2\varepsilon^4} \\ c = \dfrac{1}{\pi^2\varepsilon^2} \end{cases} \tag{27}$$

Substitute (27) to (21), get

$$y = -\frac{3}{2\pi^2\varepsilon^4}t^2 + \frac{1}{\pi^2\varepsilon^2},\ t \in [-\varepsilon, \varepsilon] \tag{28}$$

Now, a whole expression $h_4(t)$ is completed.

$$h_4(t)=\begin{cases}-\dfrac{3}{2\pi^2\varepsilon^4}t^2+\dfrac{1}{\pi^2\varepsilon^2} & |t|<\varepsilon \\ -\dfrac{1}{2\pi^2t^2} & |t|\ge\varepsilon\end{cases} \tag{29}$$

From figure 2, we can see that $h_4(t)$ is continuous at $|t|=\varepsilon$ and $t=0$ and is derivable at $t=0$ and its slickness is better than that of $h_1(t)$ and $h_2(t)$.

Let $\varepsilon=d$ and sample $h_4(t)$ using the sampling interval *d*, the practical digital filter is

$$h_4(n)=\begin{cases}-\dfrac{1}{2\pi^2n^2d^2} & n\ne0 \\ \dfrac{1}{\pi^2d^2} & n=0\end{cases} \tag{30}$$

For $h_4(t)$ of $[-\varepsilon,\varepsilon]$ is a parabola, we named the filter as parabola filter

3.2 Adaptable length discrete filter (Filter 5)

It is designed using the discrete pattern.

Considering that the ramp filter's length should be double of that of the projection signal, we can calculate $h(0)$ according to the rest values of $h(n)$ whose length is finite.

Suppose that the definition region of a ramp filter should be $[-N,N]$, $h(0)$ can be

$$h(0)=-\sum_{n\ne0}h(n)=h(-N)+h(-N+1)+\cdots+h(-1)+h(1)+\cdots+h(N-1)+h(N) \tag{31}$$

According to (11) and (31), get

$$h_5(n)=\begin{cases}-\dfrac{1}{2\pi^2n^2d^2} & n\ne0,\ n\in[-N,N] \\ -\sum\limits_{n\ne0}h(n) & n=0,\ n\in[-N,N]\end{cases} \tag{32}$$

The length of ramp filter is variational according to the projection signal, so $h_5(0)$ is changed with the change of the length of ramp filter. Thus the practical digital filter $h_5(n)$ is changed with the change of the length of ramp filter.

In the sense, we name the filter to be adaptable length discrete filter.

4　Simulation experiment for comparison of the five practical filters

Now, we have gotten 5 practical filters according to five design methods, which are (15), (18), (19), (30), (32).

They are same when $n\ne0$ and the only one different value is the central value $h(0)$ which play an important role to CT image quality.

Design a simulation model:

There is a thick cylinder which is inserted by four thin cylinders. The five cylinder's axes are parallel. Now, we will reconstruction a certain slice of the object (Figure 3). The parameters of the model are shown in Table 1.

The number of the detector's cell is 101 and its width is 0.1 cm. There are 180 projections for the angle interval is 1^{o}. The X-beam is parallel. The standard FBP algorithm is adopted. The reconstructed CT image is 101×101 and its center is the rotation center of the object.

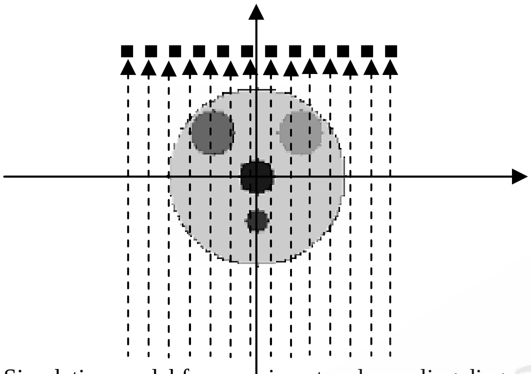

Fig.3 Simulation model for experiment and sampling diagram at 0^{o}

Table 1 Parameters of the model

Cylinder name	Parameters		
	Radius/cm	Attenuation coefficient /cm^{-1}	Coordinate
Big cylinder	4	0.8	（0，0）
Small cylinder at left and upper	1	0.4	（-2，2）
Small cylinder at right and upper	1	0.6	（2，2）
Central small cylinder	0.8	0.1	（0，0）
Lower small cylinder	0.5	0.2	（0，-2）

To evaluate the quality of the five CT images, we adopt two error criteria. One is normalization root-mean-square distance criterion *d* and the other is normalization average absolute distance criterion *r*.

Suppose that the image is $N\times N$ and that t_{uv} is the linear attenuation coefficient at line *u* and row *v* of the model image and that r_{uv} is the linear attenuation coefficient at line *u* and row *v* of the reconstructed image and $\overline{t}$ is the average attenuation coefficient of the model image. Then *d* and *r* can be expressed as (33) and (34) respectively[8].

$$d=\left(\frac{\sum_{u=1}^{N}\sum_{v=1}^{N}\left(t_{uv}-r_{uv}\right)^{2}}{\sum_{u=1}^{N}\sum_{v=1}^{N}\left(t_{uv}-\overline{t}\right)^{2}}\right)^{\frac{1}{2}} \tag{33}$$

$$r=\frac{\sum_{u=1}^{N}\sum_{v=1}^{N}\left|t_{uv}-r_{uv}\right|}{\sum_{u=1}^{N}\sum_{v=1}^{N}\left|t_{uv}\right|} \tag{34}$$

The CT images reconstructed by the five practical filters respectively are shown in Figure 4. The errors comparison is shown in Table 2.

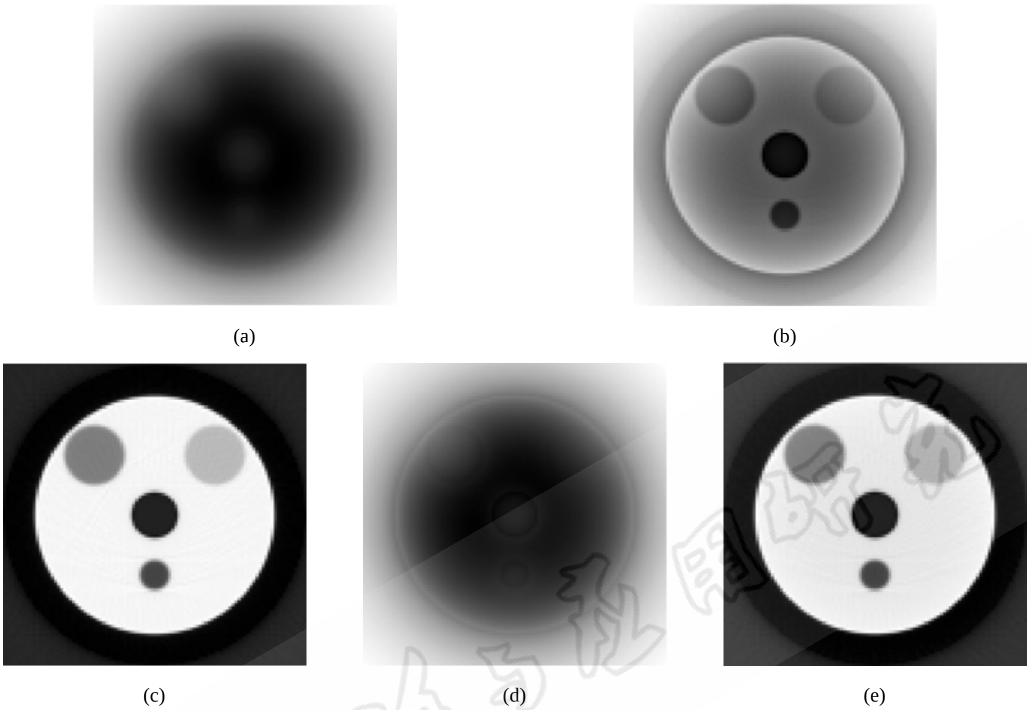

Fig.4　CT images reconstructed by five practical digital filters of the ideal ramp filter, (a) constant filter, (b) linear filter, (c) infinite length discrete filter, (d) parabola filter and (e) adaptable length discrete filter

Table 2　Errors comparison of the five practical filers

filter	Errors	
	d	*r*
1.constant filter	1 408.047	38.318
2.linear filter	22.386	4.801
3.infinite length discrete filter	0.035	0.107
4.parabola filter	446.327	21.559
5.adaptable length discrete filter	0.125	0.322

5　Results analyzing

From Figure 4 and Table 2, we can see that the image quality order is

$$\text{filter3} > \text{filter5} > \text{filter2} > \text{filter4} > \text{filter1}$$

Obviously, discrete methods are better than continuous methods.

Filter 3 is better than filter 5, showing that $h(n)$ should be regarded as an infinite signal rather than a finite signal whose length is the double of that of projection signal when we calculate the central value $h(0)$ from the discrete angle.

The frequency characteristics of $h_3(n)$ and $h_5(n)$ are shown in Figure 5.

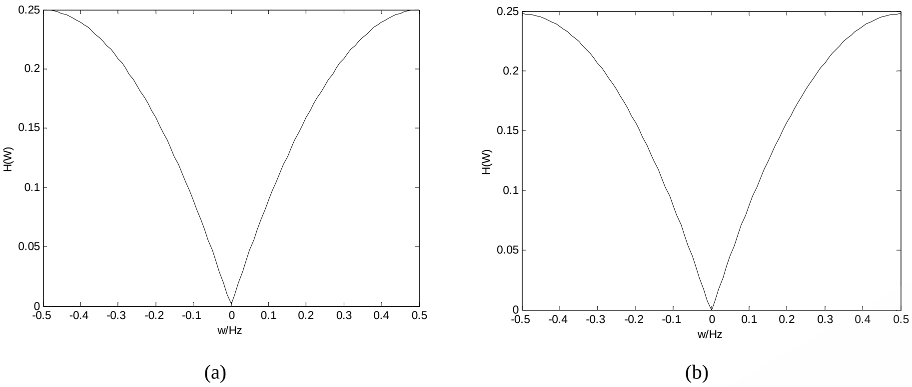

Fig.5 Frequency characteristics of filters designed form discrete angle, (a) is for $h_3(n)$ and (b) is for $h_5(n)$.

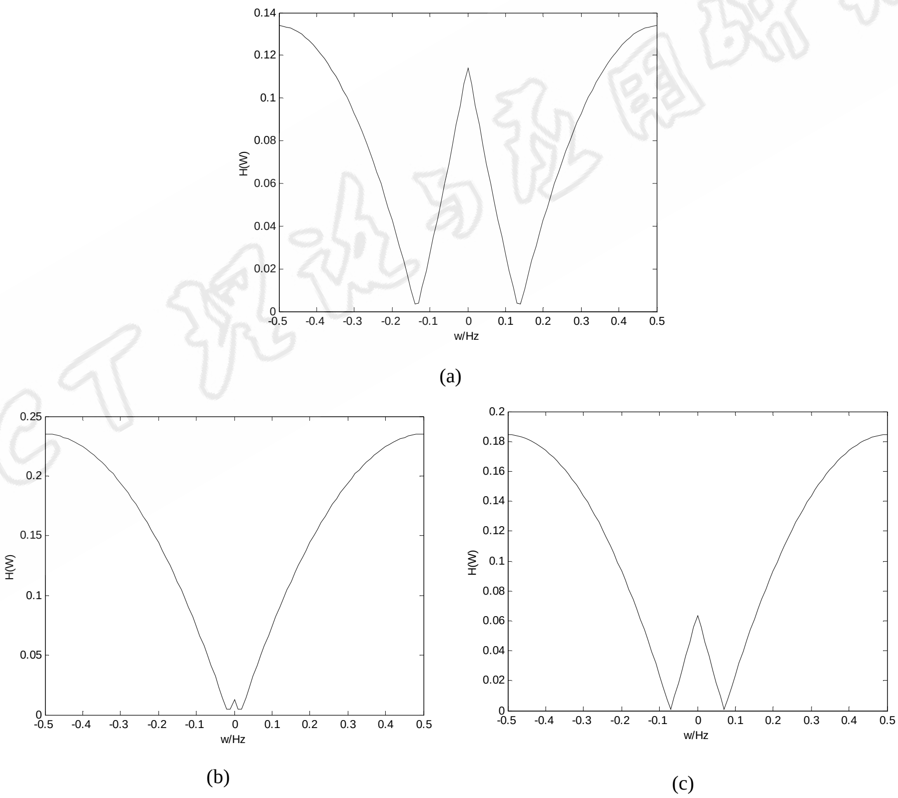

Fig.6 Frequency characteristics of filters designed from continuous angle, (a) is for $h_1(n)$ and (b) is for $h_2(n)$ and (c) is for $h_4(n)$

From Figure 5, we can see that (a) is steeper than (b) and (a) can better embody the “ramp” meaning which is nearer the standard wave of the ramp filer than (b).

They are all better than those filters designed from the continuous angle. Le us see their frequency characteristics (see Figure 6).

Obviously, their waves are all depart from the standard wave of the ramp filter.

(b) is the best one for it can embody the “ramp” meaning on a majority of region. However there is a small distortion on a small neighborhood of 0. Form Figure 4 (b), we can see that the CT image is acceptable although it is too light to express the interior structure ideally.

From (a) and (c), we can see the two filter are depart from the standard wave of the ramp filter very much. There is a big distortion on a big neighborhood of 0 in (a) and (c), which can seriously impact the reconstruction quality. So Figure 4 (a) and (d) are both two black disk. The CT images are too anamorphic to see the interior structure.

Compare Figure 6 (a) and (c), we can find that the distortion of (c) is smaller than that of (a). Meanwhile, compare Figure 4 (a) and (d), we can find that the image quality of (d) is better than that of (a). The two findings are consistent, showing that the parabola filter is better than the constant filter, which can also been explained by Table 2. In Table 2, the error of filter 4 is smaller than that of filter 1.

The linear filter is better than the constant filter and the parabola filter. When reading Wei’s paper, we may guess that if the smoother the curve of $h(t)$ on $|t|=\varepsilon$ is, the better the CT images reconstructed by using corresponding filter will be. Now we can conclude that the guess is false. The parabola is smoother than the line and it is derivable at $t=0$, but the parabola filter is not better than the linear filter, which is obvious by comparing Figure 4 (b) and (d).

6 Conclusion

The ideal ramp filter is a new and promising filter. There are two design patterns to design practical digital filters of the ideal ramp filer. One is from the continuous angle, the other is from the discrete angle. The constant filter, the linear filter and the parabola filter are all belong to the former. The infinite length discrete filter and the adaptable length discrete filter are both belong to the latter. In general, the discrete pattern is better than the continuous pattern. The filters designed using the discrete pattern can get better image quality and can be applied in engineering. In the continuous pattern filters, the linear filter is the best, which is acceptable in engineering application. The constant filter and the parabola filter cannot be applied in FBP algorithm.

More researches should be developed so as to design better practical digital filters of the ideal ramp filter.

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解析法图像重建中的理想斜变滤波器的进一步研究

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摘要：理想斜变滤波器是一种被傅里叶逆变换定义的广义函数。设计理想斜变滤波器的实用数字滤波器是最重要的任务之一。推导了理想斜变滤波器的单位冲激响应的公式。提出了两种实用数字滤波器的设计模式：连续模式和离散模式。分析了 3 种已经存在的实用滤波器，设计了 2 种新的实用滤波器。通过一个仿真实验，比较了这 5 种滤波器。总的来说，以离散模式设计的离散方法优于以连续模式设计的连续方法。对于 2 种离散方法来说，无限长离散方法优于自适应长度离散方法，它们均可以得到较好的重建质量。对于 3 种连续方法而言，线性方法是最好的，可以重建出工程可用的 CT 图像，而常数方法和抛物线方法不能重建出可用的 CT 图像。

关键词：CT；图像重建；理想斜变滤波器；滤波器设计；实用滤波器

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