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On Dose Reduction and View Number in Computed Tomography

Kaijie Lu[1,2], Baodong Liu[3,4], Hengyong Yu[5]✉

1.Department of Biomedical Engineering, Wake Forest University Health Sciences, Winston-Salem, NC 27157, USA
2.Wallace H. Coulter Department of Biomedical Engineering and Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30332, USA
3.Key Laboratory of Nuclear Analytical Techniques, Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
4.Beijing Engineering Research Center of Radiographic Techniques and Equipment, Beijing 100049, China
5.Department of Electrical and Computer Engineering, University of Massachusetts Lowell, Lowell, MA 01854, USA

Abstract: The ultimate goal in improving computed tomography (CT) technology is to reconstruct higher quality images with lower radiation dose, which can help to lower cancer risks to the patients. Inspired by the compressive sensing (CS) theory, few-view reconstruction has been a hot topic for dose reduction in recent years. However, when the radiation dose is fixed, fewer view projections does not always imply higher image quality. Numerical tests are performed in this study to investigate the relationship between image quality and view numbers under a fixed radiation dose level. It is observed that for a fixed dose level, as the view number increases, the image quality increases but then decreases once the view number is sufficiently large. For the dose level and phantom we tested, the optimal view number for best reconstruction quality is around 300 in an equi-angular full scan configuration, which provides a useful hint for practical applications.

Key words: Computed tomography; dose reduction; view number; compressive sensing; total variation minimization; ordered subset simultaneous algebraic reconstruction technique

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Computed tomography (CT) is a technology to create 2-D and 3-D images from the X-ray attenuation of the body from different angles. These images provide very important information for physicians to diagnose illnesses. In 2000, the number of CT scans was approximately 46 million, in 2006, 62 million[1], and in 2009, the number of scans was above 70 million[2]. In particular, use of pediatric CT is rapidly growing. As CT is benefiting clinical diagnosis more and more often, radiation dose associated with diagnostic CT scans has become a major concern. X-ray radiation dose may induce genetic, cancerous, and other diseases. Moreover pediatric CT

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has a potential of increased risk because developing children are much more sensitive to X-ray radiation. Reducing the risks of CT scans is one of the main bottlenecks to developing diagnostic radiology.

The filtered back projection (FBP) algorithm is currently the most widely used reconstruction technique because it is very fast and robust. The image quality of FBP-type algorithms is acceptable in most cases. However, these algorithms produce images with a large amount of noise, substantial streak artifacts, and poor low-contrast detectability in low dose situation mainly due to the back projection process[3]. The FBP technique requires a substantial number of views to reconstruct a decent image for clinical diagnosis. However, complete projection data usually require higher radiation dose.

Iterative reconstruction (IR) technique is used to overcome data insufficiency. Research has also been done to develop iterative algorithms based on algebraic reconstruction technique (ART)[4]. The simultaneous ART (SART)[5] works on the complete data and updates all the pixels to obtain a stable solution. The ordered subset version of SART (OS-SART)[6] and simultaneous iterative reconstruction technique (OS-SIRT)[7] divide the projection data into several groups and perform the update for each group instead of the complete projection data. Because the IR algorithms account for noise in the data model and use prior information, they outperform analytical approaches when the data is sparse, incomplete, and/or noisy and allow for high quality image reconstructions with reduced radiation dose. The IR works by allowing for multiple volumes to be obtained at once. This also means that IR algorithms are generally more computationally intensive[8] than the analytical counterparts. Therefore the IR methods are considered to be the best technique in reducing patient dose in practical applications including pediatric CT imaging. The main concern with IR methods is its demanding for more computation capacity because of multiple iterations. With the advance in computation facilities in the past decade, the IR is attracting more and more attention from CT researchers and manufacturers.

Recently the application of compressive sensing (CS) has become popular in signal and image processing. The CS theory shows that a high-quality signal or image can be reconstructed from far fewer measurements than what is usually required by the Nyquist sampling theorem[9-11]. The main idea of CS is that most signals or images have sparse representations in a transform domain, thus a very limited amount of samples are taken in a much less correlated basis and then the signal is recovered with an overwhelming probability from these data via l_1 minimization. Most CT images can be regarded as piece-wise constant function because the X-ray attenuation coefficient varies slightly within an anatomical component and largely around the borders of tissue structures. Therefore, the discrete gradient transform of a CT image is sparse. Inspired by the CS theory, a new CT reconstruction approach, referred to as total variation (TV, the l_1 norm of the discrete gradient transform) minimization was developed[12-15]. With the CS theory and TV method, it has attracted researchers to reconstruct higher quality images from fewer view projections for dose reduction.

In summary, reducing radiation exposure in CT is one of the ultimate goals of scientists seeking algorithms for high-quality image reconstruction. However, the process of improving the technology to reduce dose presents a dilemma because with lower dose the image quality declines. Recently, the CS-based few view reconstruction has become an increasingly popular topic. Thus some interesting questions arise. Does a smaller view number mean a better image quality for a fixed dose level? What is the optimal view number to achieve the best image quality? The purpose of this paper is to try to figure out answers to these questions. More specifically, we try to find the most optimal view number with best reconstruction quality for a fixed radiation dose level. The rest of the paper is organized as follow. In Section 2, we describe the model, projection with noise, and reconstruction algorithm. Experimental results are presented in Section 3, and conclusions and discussions are presented in Section 4.

1 Method

1.1 CT System Model

In the CT reconstruction field, a two-dimensional (2D) image f of size N_1-by-N_2 usually is rearranged as a one-dimensional (1D) vector x of length $N = N_1 \times N_2$ with X-ray attenuation coefficients at each pixel position. The model for IR technique in CT can be represented by

$$\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b} \tag{1}$$

where $\boldsymbol{b}$ represents a projection data as a 1D vector of length M and A is an M-by-N system matrix determined by the projection model and scanning geometry[16]. The goal of CT reconstruction is to solve problem (1) for $\boldsymbol{x}$.

1.2 Projection Noise Model

The CT transmission data noise arises mainly from X-ray quanta noise and system electronic noise. The transmission data can be expressed as[17],

$$I_1 = \mathrm{poisson}(\lambda) + \mathrm{Normal}(m, \sigma^2) \tag{2}$$

where the parameter λ is the expectation of the numbers of X-ray photons having traversed the patient. The second term in (2) is the normal distribution of the electronic noise with mean m and variance σ^2. In practice, m is often calibrated to be zero. According to the Lambert-Beer's law, the projection integral p can be approximately defined as the negative logarithm of the ratio between the outgoing and incoming number of photons,

$$p = -\ln\left(\frac{I_1}{I_0}\right) = \ln(I_0) - \ln(I_1) \tag{3}$$

The expectation λ in (2) can be expressed as,

$$\lambda = I_0 \exp(-\overline{p}) \tag{4}$$

where $-\overline{p}$ is the expectation of p.

In this work, a monochromatic X-ray is assumed. A noise-free linear-integral measurement $\overline{p}$ is computed using a given image phantom in an equiangular fan beam geometry. For a given I_0, the transmission data I_1 can be simulated using (2) and (4). Finally, the noisy data p is calculated using (3) and the projection noise

$$n = p - \overline{p} \tag{5}$$

It should be pointed out that the above process is used to generate one measurement at one detector cell and one view angle, which can be viewed as one component of $\boldsymbol{b}$ in Eq.(1). The parameter I_0 in (3) is proportional to the X-ray photon intensity prior to arrival at the body. According to the table III and the relationship $\Gamma = \frac{1}{I_0}$ in[17], we can establish a relationship between I_0 and mAs level for a fixed view number (the correspondence between I_0 and mAs is given in Section 3), hence the dose level. Because the dose level is proportional to the product of I_0 and view number for a scanning geometry and detector configuration, this product is used to represent a measurement of dose level in this work.

1.3 Reconstruction Method

Suppose that x^k is the current approximation to a solution of system (1) after k iterations. The SART-type solution to system (1) can be written as[6],

$$x_j^{k+1} = x_j^k + \lambda_k \frac{1}{\sum_{i=1}^{M} a_{i,j}} \sum_{i=1}^{M} \left(\frac{b_l - \left\langle a^i, x^k \right\rangle}{\sum_{i=1}^{M} a_{i,j}} a_{i,j} \right) \tag{6}$$

where $\left\langle a^i, x^k \right\rangle$ is the inner product of the i^{th} row a^i of $\boldsymbol{A}$ and x^k, a_{ij} is the j^{th} entry of the row a^i, $\lambda_k \in (0, 2)$ is the relaxation parameter, and the index i is cyclically taken as 1, 2, $\cdots$.

The OS-SART technique divides projection data into several sets. In each iteration, the SART algorithm is applied to each individual subset, and the intermediate reconstructed image is used as the input to the next subset reconstruction until passing through all of the subsets. Inspired by the CS theory, the TV minimization method is adopted to reconstruct the image f by solving the following optimization problem[18],

$$\min \mathrm{TV}(f) \text{ subject to } \boldsymbol{Ax} = \boldsymbol{b}, \tag{7}$$

where

$$\mathrm{TV}(f) = \sum_{i-1}^{I} \sum_{j=1}^{J} d_{i,j}$$

and

$$d_{i,j} = \left(\left(f_{i,j} - f_{i+1,j} \right)^2 + \left(f_{i,j} - f_{i,j+1} \right)^2 \right)^{\frac{1}{2}}$$

In this paper, we employ the classical alternative minimization method to minimize Eq.(7). While the SART or OS-SART is used to minimize the data discrepancy to satisfy the projection constraint $\boldsymbol{Ax} = \boldsymbol{b}$ in the first step, the steepest descent search method is used to minimize the TV[13].

2 Experimental Results

To investigate the relationship between the image quality and the view numbers for a fixed radiation dose, numerical tests were performed on different dose levels and view numbers in MatLab. A 512×512 image of a patient's chest was selected as a digital image phantom. Typical fan beam geometry with a circular scanning locus was used. The radius of the field of view was 25.0 cm. The detector array contains 888 elements. The distance between the source and object is 53.852 cm and object to detector is 49.83 cm. For each of the selected photon number and view number over a full scan range, we first acquired the projection dataset with noise as explained in section 2.2. The parameters for the electronic noise are $m = 0$ and $\sigma^2 = 10$[17]. Then we reconstructed the images using OS-SART with TV minimization algorithm. The parameter λ_k in the OS-SART iteration was set to be a constant 1.0, and the maximum number of iterations performed was calculated by (40 000/view number).

In our experiments, the numbers of views N_{vigw} were taken 10, 20, 30, 40, 50, 80, 110, 160, 210, 290, 400, 570, 790, 1 110, and 1 550; the total number of photons K (the product of the view number and I_0) was taken as 4 different values of 5×10^6, 10^7, 2×10^7 and 5×10^7. Because K is proportional to the dose level, here we will use K to indicate the radiation dose level. Based on the datasets in Table III in[17], a linear relationship between the total number of K and the dose level was fitted as $K=1.084\times10^6 d$, where d is the dose level (mAs). For illustration, the values of photons, I_0 and mAs level for the cases of view numbers = 50, 290, and 1 550 are shown in Table 1. Figure 1 shows the cardiac region of the reconstructed images for different cases. It should be pointed out that we did not consider the bowtie filter in our simulations.

Table 1　The dose level and I_0 for different simulation cases

Nview	K = 5 e6 4.61 mAs	K = 1 e7 9.22 mAs	K = 2 e7 18.45 mAs	K = 5 e7 46.13 mAs
50	1.0 e5	2.0 e5	4.0 e5	1.0 e6
290	1.7 e4	3.4 e4	6.9 e4	1.7 e5
1 550	0.3 e4	0.6 e4	1.3 e4	3.2 e4

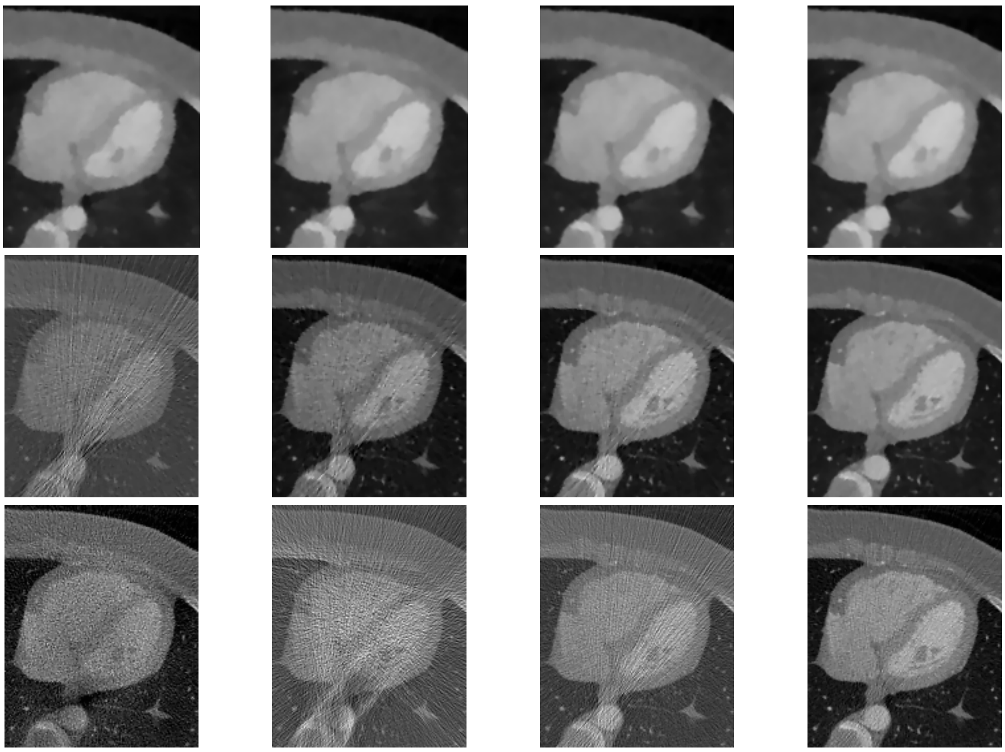

Fig.1　The cardiac region of the reconstructed images with different dose and view numbers. From left to right, the dose levels are 4.61 mAs, 9.22 mAs, 18.45 mAs and 46.13 mAs, respectively. From top to bottom, the view numbers are 50, 290 and 1 550, respectively. While the maximum HU number is mapped to white, the minimum HU number is mapped to black

Each reconstructed image $F_{N_1 \times N_2}$ was compared with the original image $f_{N_1 \times N_2}$ using the root mean squared error.

$$\mathrm{RMSE} = \frac{\left(\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \left(f_{ij} - F_{ij} \right)^2 \right)^{\frac{1}{2}}}{N_1 \times N_2}$$

Figure 2 shows the RMSE comparison of the reconstructed images with different dose levels and views. For a fixed dose level, as the number of views increases, RMSE drops to a certain point but then rises up, which indicates that the image quality changes towards good but then towards bad again. This is because larger view number brings more information but more noise while smaller view number brings less noise but less information. Meanwhile, electronic noise will compromise the image quality when I_0 becomes smaller with the increase of the view number. Hence, there is a trade-off between the view number and noise. We also compared the qualities of the images using the structural similarity index (SSIM)[19], which is a non-unit value between 0 and 1. With the original phantom image as reference, the greater the SSIM, the better the image quality. Figure 3 shows the SSIM comparison of the same settings as in Figure 2. It shows a similar trend to RMSE. Note that the log in Figures 2 and 3 is natural logarithm.

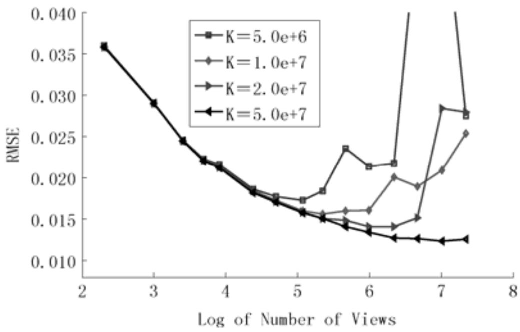

Fig.2 RMSE vs. view numbers

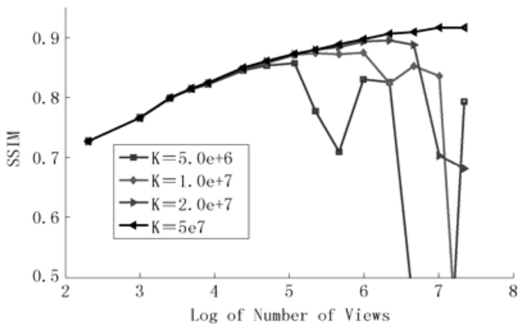

Fig. 3 SSIM vs. view numbers

From the experimental results, for different dose level K, it is observed that, for a fixed view number, larger K gives lower RMSE and higher SSIM, thus better image quality. As K increases, the minimum RMSE and the maximum SSIM increase as well, which implies the capacity of image quality is better. This is consistent with our common sense in the CT community. More interestingly and importantly, the best image quality was attained at around 300-400 views for different dose level we have tested. Views less than this range will result in lack of information even though the noise is less; on the other hand, views more than that will bring too much noise to compromise the additional measurements. For the case of 5×10^7 photons, the image quality does not show decay before 1 550 views. This can be explained by the relationship between higher photon number and Poisson noise. the higher the photon the number K, the smaller the Poisson noise will be, which yield a smaller total noise given the fact that the electronic noise is fixed.

3 Discussion and Conclusion

Reducing the number of views is important to lower radiation dose; however, the view numbers can only be reduced by so much before it begins to affect the image quality and thus undermining the purpose of the CT scan. For a fixed dose level, fewer view number may imply inferior image quality. Therefore, there is a trade-off between view number and the image quality for a fixed dose level. General speaking, larger view number brings more information but more noise while smaller view number brings less noise but less information. It is necessary to investigate the direct relationship between the image quality and view number for a fixed dose level.

In this work, we have performed several preliminary numerical tests to compare the image qualities with different dose level and view numbers. It is observed that for a fixed dose level, as the view number increases, the image quality increases but then decreases. For the cases we tested, the optimal view number for best image quality is around 300. For a given sparse transform, it should be pointed out that the optimal view number definitely depend on the image contents (e.g. unknown variables in the reconstructed images), as well as detector resolution (e.g. the detector cell number). Therefore, the optimal view number 300 is only suitable for the tested image in this paper, and it cannot be directly applied to other applications. Nevertheless, the reported results should provide a useful hint for clinical applications given the fact the image phantom is a representative cardiac image.

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CT 辐射量降低和投影数目研究

Kaijie Lu[1, 2], 刘宝东[3, 4], Hengyong Yu[5]✉

1. Department of Biomedical Engineering, Wake Forest University Health Sciences, Winston-Salem, NC 27157, USA
2. Department of Biomedical Engineering, Scheller College of Business, Georgia Institute of Technology, Atlanta, GA 30332, USA
3. 中国科学院高能物理研究所核分析技术重点实验室，北京 100049
4. 北京市射线成像技术与装备工程技术研究中心，北京 100049
5. Department of Electrical and Computer Engineering, University of Massachusetts Lowell, Lowell, MA 01854, USA

摘要：改进 CT 技术的最终目标是用较低的辐射剂量重建出更高质量的图像以降低患癌症的风险。近年来受压缩感知理论的启发，减少投影角度重建一直是减少辐射量的一个热门课题。但是，当辐射剂量固定，减少投影角度并不总是意味着更好的图像质量。本文研究固定辐射剂量下图像质量和扫描角度数目的关系。数值实验表明对于固定的辐射剂量，起初图像质量随着扫描角度数目的增加而提高，但是当扫描角度数目足够大之后图像质量反而下降了。在等角全扫描模式下，对于我们测试的辐射剂量和图像，产生最佳图像质量的最佳扫描角度数目是 300，这对于实际的应用具有借鉴意义.

关键词：CT；辐射剂量减少；扫描角度数目；压缩感知；全变差最小化；OS-SART

Biography: Kaijie Lu is a Senior at Georgia Tech Scheller College of Business and College of Engineering. He was Research Assistant with the Department of Biomedical Engineering, Wake Forest University Health Sciences (2013 Summer), and Math Tutor with Georgia Southern University (2010~2011). Hengyong Yu✉ (email: hengyong-yu@ieee.org) received his Bachelor degrees in information science & technology (1998) and computational mathematics (1998), and his PhD in information & telecommunication engineering (2003) from Xi'an Jiaotong University. He was Associate Research Scientist with the University of Iowa (2004～2006), Research Scientist with Virginia Tech (2006～2010), Assistant Professor with Wake Forest University Health Sciences (2010-2014). Since 2014, he has been an Associate Professor, the Director of Imaging and Informatics Lab, Department of Electrical and Computer Engineering, University of Massachusetts Lowell. His interests include computed tomography and medical image processing. He has authored or coauthored > 110 peer-reviewed journal papers. He is a senior member of the Institute of Electrical and Electronics Engineers (IEEE) and the IEEE Engineering in Medicine and Biology Society (EMBS), and member of the American Association of Physicists in Medicine (AAPM) and the Biomedical Engineering Society (BMES). In 2012, he received an NSF CAREER award for the development of compressive sensing based interior tomography.